Finals Individual Test

Congrats, you made it to the final round, meaning you already placed top at your category. You have 40 minutes to complete this test. Don't expect to answer all of them. Show your work, because partial credit will be awarded. Questions 1-5 are worth 6 points. Questions 6-8 are worth 10 points.

1. Two eight-sided dice are rolled, and a number appears from 1 to 8 with equal probability. What is the probability the product of the two numbers is greater than their sum?
2. In the United States, poker chips are the following thicknesses: blues are 1.55 mm, reds are 1.95 mm, greens are 1.35 mm, and blacks are 1.75 mm. Jimmy wins a big pot, and his stack is 14 mm high. How many chips did he win?
3. The product of three consecutive integers is 8 times their sum. What is the sum of their squares?
4. Find the value(s) of *x* such that $8xy - 12y + 2x - 3 = 0$ is true for all values of *y*.
5. A circle passes through the three vertices of an isosceles triangle that has two sides of length 3 and a base of length 2. What is the area of this circle?
6. A regular octagon ABCDEFGH has sides of length two. Find the area of triangle ADG.
7. Let *a, b* and *c* be real numbers such that $a - 7b + 8c = 4$and $8a + 4b - c = 7$. Then $a^2 - b^2 + c^2$ is what?
8. Let n denote the smallest positive integer that is divisible by both 4 and 9 and whose base-10 representation consists of only 4's and 9's, with at least one of each. What are the last four digits of n?